A Semantics for Means-End Ascriptions

Jesse Hughes

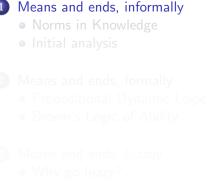
Technical University of Eindhoven

September 21, 2004

<ロ> (四) (四) (三) (三) (三)

-2

Outline



Fuzzy sets

(日) (日) (日) (日) (日)

Outline

Means and ends, informally

 Norms in Knowledge
 Initial analysis

 Means and ends, formally

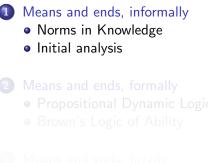
 Propositional Dynamic Logi
 Brown's Logic of Ability

 Means and ends, fuzzily

- Why go fuzzy?
- Fuzzy sets

< □ > < □ > < □ >

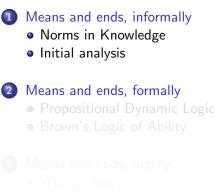
Outline



- Why go fuzzy?
 - Fuzzy sets

<回> < 回> < 回> < 回>

Outline



Fuzzy sets

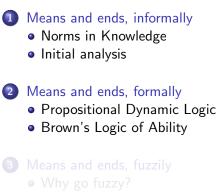
<回> < 回> < 回> < 回>

Outline



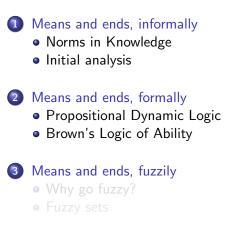
・ 同 ト・ ・ ヨート・ ・ ヨート

Outline



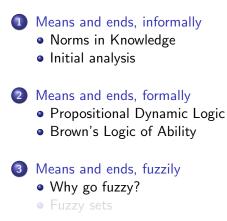
Fuzzy sets

Outline

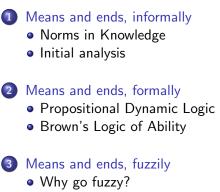


• E > •

Outline

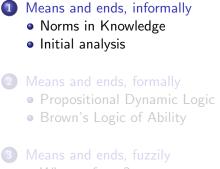


Outline

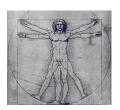


• Fuzzy sets

Outline



- Why go fuzzy?
- Fuzzy sets



Epistemology:

- Knowledge of descriptive claims
- Knowledge of normative claims
 - Non-moral



Epistemology:

- Knowledge of descriptive claims
- Knowledge of normative claims
 - Non-moral
 - Prescriptive ought to do
 - Functional --- things ought to do

(日) (日) (日) (日) (日)

æ



Epistemology:

- Knowledge of descriptive claims
- Knowledge of normative claims
 - Non-moral
 - Prescriptive ought to do
 - Functional things ought to do

イロト イヨト イヨト イヨト

Arbitacts: arbitactual functions



Epistemology:

- Knowledge of descriptive claims
- Knowledge of normative claims
 - Non-moral
 - Prescriptive ought to do Artifacts: HOWTOs
 - Functional things ought to do

Artifacts: artifactual functions



Epistemology:

- Knowledge of descriptive claims
- Knowledge of normative claims
 - Non-moral
 - Prescriptive ought to do Artifacts: HOWTOs
 - Functional things ought to do Artifacts: artifactual functions

Norms in Knowledge Initial analysis

Introduction to Norms in Knowledge An epistemological investigation.



Applied to technical artifacts:

- Knowledge of descriptive claims
- Knowledge of normative claims
 - Non-moral
 - Prescriptive ought to do Artifacts: HOWTOs
 - Functional things ought to do Artifacts: artifactual functions

Norms in Knowledge Initial analysis

Introduction to Norms in Knowledge An epistemological investigation.



Applied to technical artifacts:

• Knowledge of descriptive claims

- Knowledge of normative claims
 - Non-moral
 - Prescriptive ought to do Artifacts: HOWTOs
 - Functional things ought to do

Artifacts: artifactual functions

Norms in Knowledge Initial analysis

Introduction to Norms in Knowledge An epistemological investigation.



Applied to technical artifacts:

• Knowledge of descriptive claims

- Knowledge of normative claims
 - Non-moral
 - Prescriptive ought to do Artifacts: HOWTOs
 - Functional things ought to do Artifacts: artifactual functions

Norms in Knowledge Initial analysis

Some examples of functional ascriptions



• "The function of the heart is to pump blood."

- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff,

・ロト ・ 同ト ・ ヨト ・ ヨト



- "The function of the heart is to pump blood."
- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff,



- "The function of the heart is to pump blood."
- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff, and the algorithms



- "The function of the heart is to pump blood."
- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff, artifacts, algorithms,

< ロト (周) (三) (三)



- "The function of the heart is to pump blood."
- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff, artifacts, algorithms, personal roles...



- "The function of the heart is to pump blood."
- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff, artifacts, algorithms, personal roles...



- "The function of the heart is to pump blood."
- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff, artifacts, algorithms, personal roles...



- "The function of the heart is to pump blood."
- "That switch mutes the television."
- "The subroutine ensures that the user is authorized."
- "The magician's assistant is for distracting the audience."

We ascribe functions to biological stuff, artifacts, algorithms, personal roles...

"That switch mutes the television."

One can use the switch to mute the television. There is an action involving the switch that will cause the television to be muted.

• Functions imply means-end relations.

• Step one: Provide a semantics for means-end relations.

(4月) (1日) (日)

"That switch mutes the television." \Downarrow One can use the switch to mute the television. \Downarrow

There is an action involving the switch that will cause the television to be muted.

• Functions imply means-end relations.

• Step one: Provide a semantics for means-end relations.

"That switch mutes the television." \downarrow One can use the switch to mute the television. \downarrow There is an action involving the switch that will cause the television to be muted.

• Functions imply means-end relations.

• Step one: Provide a semantics for means-end relations.

• Functions imply means-end relations.

• Step one: Provide a semantics for means-end relations.

- Functions imply means-end relations.
- Step one: Provide a semantics for means-end relations.

・同・ ・ヨ・ ・ヨ・

• Why not?

- Formal semantics are artificial.
- Too simple or bloody complicated.
- Or both.

• You've convinced me... Can we go?



- 0
- .

イロト イヨト イヨト イヨト

4

• Why not?

• Formal semantics are artificial.

- Too simple or bloody complicated.
- Or both.

• You've convinced me... Can we go?



0

イロト イヨト イヨト イヨト

4

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me...Can we go?
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly)....
 - In our project description.

イロト イヨト イヨト イヨト

-2

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me... Can we go?
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly) ...
 - In our project description...

(日) (日) (日) (日) (日)

-2

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me... Can we go?
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly)
 - In our project description.

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me...Can we go? No.
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly) ...
 - In our project description.

< ロト (周) (日) (日)

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me...Can we go? No.
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly) ...
 - In our project description.

・ 同 ト・ ・ ヨート・ ・ ヨート

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me...Can we go? No.
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly) ...
 - In our project description.

A (1) > A (2) > A

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me...Can we go? No.
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly) ...
 - In our project description.

▲□ ▶ ▲ ■ ▶ ▲

- Why not?
 - Formal semantics are artificial.
 - Too simple or bloody complicated.
 - Or both.
- You've convinced me...Can we go? No.
 - Formal semantics provide precise claims.
 - Consequences are clear, indisputable.
 - Yield rules of inference and (importantly) ...
 - In our project description.

▲□ ▶ ▲ ■ ▶ ▲

• An end is some desirable condition.

- A means is a way of making the end true.
- Means change things: means are *actions*.

Some controversies.

- Ends-in-themselves?
- Objects as means?

イロト イヨト イヨト イヨト

-2

- An end is some desirable condition.
- A means is a way of making the end true.
- Means change things: means are *actions*.

Some controversies.

- Ends-in-themselves?
- Objects as means?

イロト イヨト イヨト イヨト

-2

- An end is some desirable condition.
- A means is a way of making the end true.
- Means change things: means are actions.

Some controversies.

- Ends-in-themselves?
- Objects as means?

- An end is some desirable condition.
- A means is a way of making the end true.
- Means change things: means are actions.

Some controversies.

- Ends-in-themselves?
- Objects as means?

イロト イポト イヨト イヨト

- An end is some desirable condition.
- A means is a way of making the end true.
- Means change things: means are actions.

Some controversies.

- Ends-in-themselves?
- Objects as means?

イロト イポト イヨト イヨト

- An end is some desirable condition.
- A means is a way of making the end true.
- Means change things: means are actions.

Some controversies.

- Ends-in-themselves?
- Objects as means?

★問▶ ★注▶ ★注▶

Possible worlds and making propositions come true

Ends are propositions we want to make true.

But actions don't change the meaning of propositions.

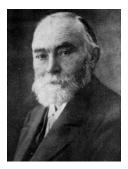
Think of a set of possible worlds.

At each time, one world is the actual world.

And at each world, every proposition is true or false.

Norms in Knowledge Initial analysis

Possible worlds and making propositions come true



Ends are propositions we want to make true.

But actions don't change the meaning of propositions.

Think of a set of possible worlds.

At each time, one world is the actual world.

And at each world, every proposition is true or false.

Norms in Knowledge Initial analysis

Possible worlds and making propositions come true



Ends are propositions we want to make true.

But actions don't change the meaning of propositions.

Think of a set of possible worlds.

At each time, *one* world is the actual world. And at each world, every proposition is true

Norms in Knowledge Initial analysis

Possible worlds and making propositions come true



Ends are propositions we want to make true.

But actions don't change the meaning of propositions.

Think of a set of possible worlds.

At each time, one world is the actual world.

And at each world, every proposition is true or false.

▲□→ ▲ □→ ▲

Norms in Knowledge Initial analysis

Possible worlds and making propositions come true



Ends are propositions we want to make true.

But actions don't change the meaning of propositions.

Think of a set of possible worlds.

At each time, one world is the actual world.

And at each world, every proposition is true or false.

A (1) > (1) > (1)

Outline

Means and ends, informally
 Norms in Knowledge
 Initial analysis

Means and ends, formally
 Propositional Dynamic Logic
 Brown's Logic of Ability

- 3 Means and ends, fuzzily
 - Why go fuzzy?
 - Fuzzy sets

・ 同 ト・ ・ ヨート・ ・ ヨート



A set of worlds involving a footrace and starter pistol.

• Two basic properties:

- Footrace started?
- Pistol loaded?

< ロト (周) (日) (日)

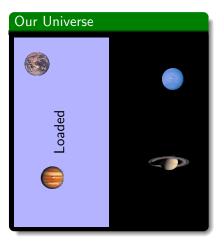


A set of worlds involving a footrace and starter pistol.

- Two basic properties:
 - Footrace started?

▲□ ▶ ▲ ■ ▶ ▲

• Pistol loaded?



A set of worlds involving a footrace and starter pistol.

- Two basic properties:
 - Footrace started?

A (1) > A (2) > A

• Pistol loaded?

Our Univers	e		
	Star	ted	
-oaded			
) Loa			
			Ū

A set of worlds involving a footrace and starter pistol.

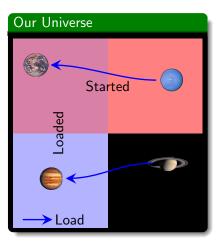
- Two basic properties:
 - Footrace started?

▲□ ▶ ▲ ■ ▶ ▲

• Pistol loaded?

Propositional Dynamic Logic Brown's Logic of Ability

A simple example of possible worlds



- Two basic actions:
 - Loading the pistol
 - Firing the pistol

・ロト ・日下・ ・ヨト・・

3.0

Propositional Dynamic Logic Brown's Logic of Ability

A simple example of possible worlds

Our Universe		
Star	rted	
-oaded		
D Foa		
→ Load	→ Fire	

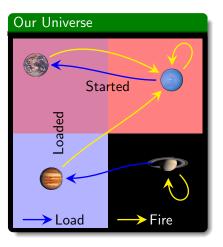
- Two basic actions:
 - Loading the pistol

・ 同・ ・ ヨ・・・・

• Firing the pistol

Propositional Dynamic Logic Brown's Logic of Ability

A simple example of possible worlds

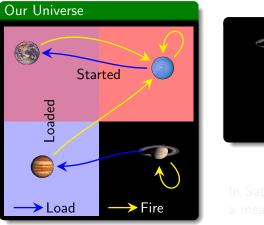


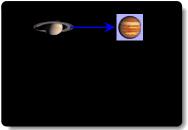
- Two basic actions:
 - Loading the pistol
 - Firing the pistol

-

Propositional Dynamic Logic Brown's Logic of Ability

A simple example of possible worlds

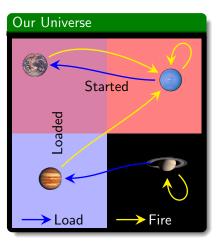


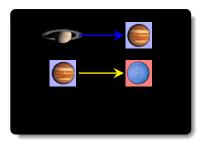


In Saturn, loading and firing is a means to starting the race.

Propositional Dynamic Logic Brown's Logic of Ability

A simple example of possible worlds

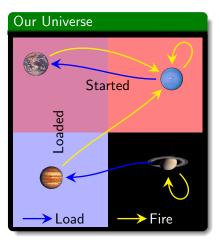


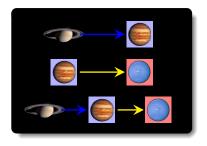


In Saturn, loading and firing is a means to starting the race.

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples

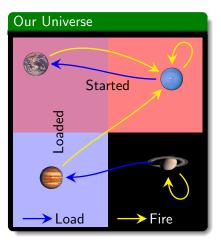


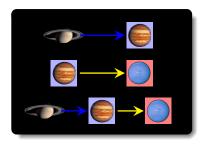


In Saturn, loading and firing is a means to starting the race.

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples

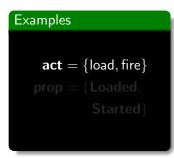




In Saturn, loading and firing is a means to starting the race.

Propositional Dynamic Logic Brown's Logic of Ability

Introducing the formal language PDL Propositional dynamic logic



- Basic ingredients:
 - A set **act** of actions
 - A set **prop** of propositions

• Action constructions:

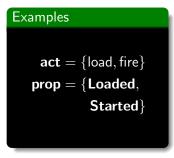
- For building complex actions
- Composition, iteration, choice, test

・ロト ・ 日 ・ ・ ヨ ・ ・ モ ト

• Formula constructions:

Propositional Dynamic Logic Brown's Logic of Ability

Introducing the formal language PDL Propositional dynamic logic



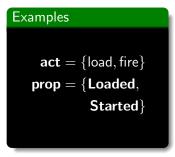
- Basic ingredients:
 - A set act of actions
 - A set **prop** of propositions
- Action constructions:
 - For building complex actions
 - Composition, iteration, choice, test

(日) (日) (日) (日) (日)

- Formula constructions:
 - Boolean connectives

Propositional Dynamic Logic Brown's Logic of Ability

Introducing the formal language PDL Propositional dynamic logic



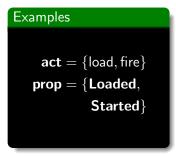
- Basic ingredients:
 - A set act of actions
 - A set **prop** of propositions
- Action constructions:
 - For building complex actions
 - Composition, iteration, choice, test
- Formula constructions:
 - Boolean connectives

 - (m)φ weak dynamic operator

< ロト (周) (日) (日)

Propositional Dynamic Logic Brown's Logic of Ability

Introducing the formal language PDL Propositional dynamic logic

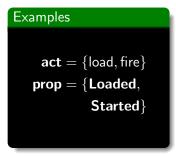


- Basic ingredients:
 - A set act of actions
 - A set **prop** of propositions
- Action constructions:
 - For building complex actions
 - Composition, iteration, choice, test
- Formula constructions:
 - Boolean connectives
 - $[m]\varphi$ strong dynamic operator
 - (m)φ weak dynamic operator

< ロト (周) (日) (日)

Propositional Dynamic Logic Brown's Logic of Ability

Introducing the formal language PDL Propositional dynamic logic



- Basic ingredients:
 - A set act of actions
 - A set **prop** of propositions
- Action constructions:
 - For building complex actions
 - Composition, iteration, choice, test
- Formula constructions:
 - Boolean connectives
 - [m]φ strong dynamic operator
 ⟨m⟩φ weak dynamic operator

・ロト ・ 日 ・ ・ ヨ ・ ・ モ ト

Propositional Dynamic Logic Brown's Logic of Ability

Introducing the formal language PDL Propositional dynamic logic

Dynamic operators

 $\begin{array}{l} [m] \varphi: \\ m \ will \ \text{bring about } \varphi. \\ \langle m \rangle \varphi: \\ m \ can \ \text{bring about } \varphi. \end{array}$

- Basic ingredients:
 - A set act of actions
 - A set **prop** of propositions
- Action constructions:
 - For building complex actions
 - Composition, iteration, choice, test
- Formula constructions:
 - Boolean connectives
 - $[m]\varphi$ strong dynamic operator
 - $\langle m \rangle \varphi$ weak dynamic operator

・ロト ・ 日 ・ ・ ヨ ・ ・ モ ト ・

Propositional Dynamic Logic Brown's Logic of Ability

Introducing the formal language PDL Propositional dynamic logic

Dynamic operators

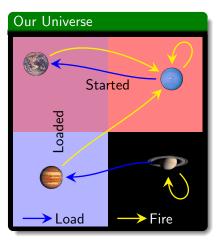
 $\begin{array}{l} [m] \varphi: \\ m \ \ will \ \ bring \ \ about \ \ \varphi. \\ \langle m \rangle \varphi: \\ m \ \ can \ \ bring \ \ about \ \ \varphi. \end{array}$

- Basic ingredients:
 - A set act of actions
 - A set **prop** of propositions
- Action constructions:
 - For building complex actions
 - Composition, iteration, choice, test
- Formula constructions:
 - Boolean connectives
 - $[m]\varphi$ strong dynamic operator
 - $\langle m \rangle \varphi$ weak dynamic operator

(D) (A) (A)

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



[fire]Started

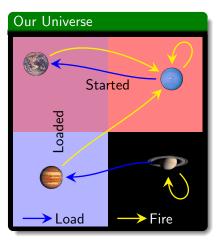
fire *will* make **Started** true. True:

(日) (部) (注) (注)

4

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



[fire]Started

fire will make **Started** true.

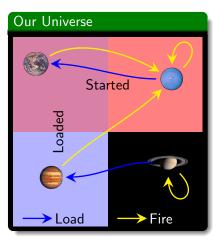
(日) (部) (注) (注)

4

True:

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



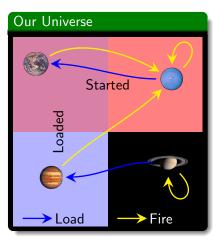


(日) (部) (注) (注)

4

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



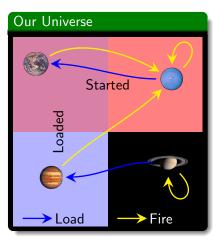
[fire]**Started** fire *will* make **Started** true. True:

(日) (部) (注) (注)

4

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



[fire]Started

fire *will* make **Started** true. True:

(日) (部) (注) (注)

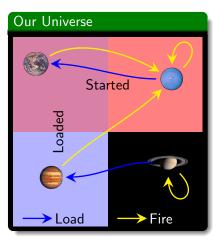
4

False:

Jesse Hughes A Semantics for Means-End Ascriptions

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



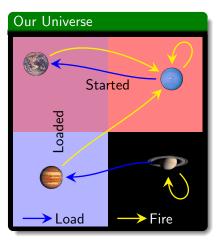


(日) (部) (注) (注)

æ

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



$\label{eq:load} $$ \label{eq:load} $$ \label{eq:l$

True:

False:

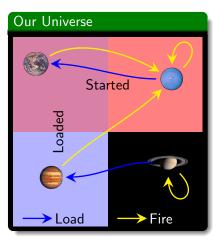
But *every* world satisfies [load]**Loaded**!

(日) (部) (注) (注)

4

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



(load) Loaded
load can make Loaded true.
 True:
 False:

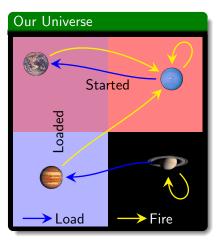
But *every* world satisfies [load]**Loaded**!

(日) (部) (注) (注)

æ

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



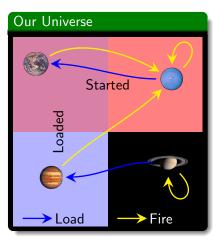
{load}Loaded load can make Loaded true. True: False: But every world satisfies

(日) (部) (注) (注)

Jesse Hughes A Semantics for Means-End Ascriptions

Propositional Dynamic Logic Brown's Logic of Ability

Some PDL examples



(load) Loaded
load can make Loaded true.
True:

False:



(日) (部) (注) (注)

But *every* world satisfies [load]**Loaded**!

Actions may have uncertain outcomes.

- Randomness
- Uncertain conditions
- Actions may require skill
- Malfunctioning artifacts

Our models should support non-determinism.

イロト イヨト イヨト イヨト



Actions may have uncertain outcomes.

• Randomness

- Uncertain conditions
- Actions may require skill
- Malfunctioning artifacts

Our models should support *non-determinism*.



Actions may have uncertain outcomes.

- Randomness
- Uncertain conditions
- Actions may require skill
- Malfunctioning artifacts

Our models should support *non-determinism*.



Actions may have uncertain outcomes.

- Randomness
- Uncertain conditions
- Actions may require skill
- Malfunctioning artifacts

Our models should support *non-determinism*.

< ロト (周) (日) (日)



Actions may have uncertain outcomes.

- Randomness
- Uncertain conditions
- Actions may require skill
- Malfunctioning artifacts

Our models should support *non-determinism*.

イロト イポト イヨト イヨト

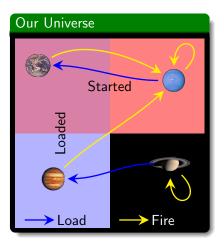


Actions may have uncertain outcomes.

- Randomness
- Uncertain conditions
- Actions may require skill
- Malfunctioning artifacts

Our models should support non-determinism.

・ 同 ト・ ・ ヨート・ ・ ヨート



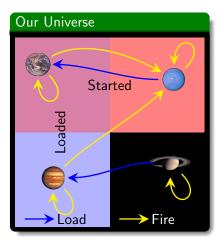
The pistol has a weak spring. Sometimes, bullet doesn't fire, world doesn't change.

$\llbracket \mathsf{fire} \rrbracket \begin{pmatrix} & \\ & \end{pmatrix} = \left\{ & & \\ & , & \\ & \\ & \end{pmatrix}.$

Actions take a world to a *set* of worlds!

 $\llbracket \mathsf{fire} \rrbracket : \mathcal{W} \to \mathcal{PW}$

イロト イヨト イヨト イヨト

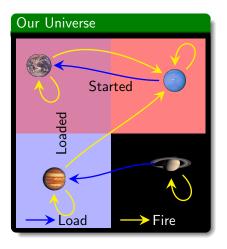


The pistol has a weak spring. Sometimes, bullet doesn't fire, world doesn't change.

Actions take a world to a *set* of worlds!

 $\llbracket \mathsf{fire} \rrbracket : \mathcal{W} \to \mathcal{PW}$

(日) (日) (日) (日) (日)



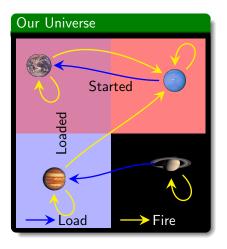
The pistol has a weak spring. Sometimes, bullet doesn't fire, world doesn't change.

$$\llbracket \mathsf{fire} \rrbracket \left(\bigcirc \right) = \left\{ \bigcirc, \bigcirc \right\}.$$

Actions take a world to a *set* of worlds!

 $\llbracket \mathsf{fire} \rrbracket : \mathcal{W} \to \mathcal{PW}$

イロト イポト イヨト イヨト



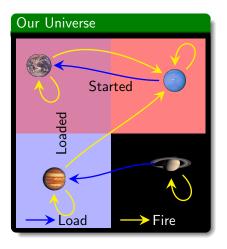
The pistol has a weak spring. Sometimes, bullet doesn't fire, world doesn't change.

$$\llbracket \mathsf{fire} \rrbracket \left(\bigcirc \right) = \left\{ \bigcirc \right) = \left\{ \bigcirc \right\}$$
.

Actions take a world to a *set* of worlds!

 $\llbracket \mathsf{fire} \rrbracket : \mathcal{W} \to \mathcal{PW}$

イロト イポト イヨト イヨト



The pistol has a weak spring. Sometimes, bullet doesn't fire, world doesn't change.

$$\llbracket \mathsf{fire} \rrbracket \left(\bigcirc \right) = \left\{ \bigcirc \right) = \left\{ \bigcirc \right\}.$$

Actions take a world to a *set* of worlds!

 $\llbracket \mathsf{fire} \rrbracket : \mathcal{W} \to \mathcal{PW}$

< ロト (周) (日) (日)

Ability and modal logic: Kenny's analysis

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).

 $\mathbf{1} \not\models \varphi \to \operatorname{Can} \varphi$

イロン イヨン イヨン イヨン

-2

Ability and modal logic: Kenny's analysis

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).

 I hit the bull, but I am not able to hit the bull.

<ロ> (四) (四) (三) (三) (三)

-2

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).

$\bullet \not\models \varphi \to \operatorname{Can} \varphi$

• I hit the bull, but I am not *able* to hit the bull.

イロト イヨト イヨト イヨト

-2

$$\textcircled{2} \hspace{0.1in} \not\models \hspace{0.1in} {\sf Can} \hspace{0.1in} (\varphi \lor \psi) \to ({\sf Can} \hspace{0.1in} \varphi \lor {\sf Can} \hspace{0.1in} \psi)$$

 I can hit bottom or top, but NOT (I can hit bottom or I can hit top).

Ability and modal logic: Kenny's analysis

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).



$$\mathbf{0} \not\models \varphi \to \operatorname{\mathsf{Can}} \varphi$$

• I hit the bull, but I am not *able* to hit the bull.

イロト イポト イヨト イヨト

$$() \not\models \mathsf{Can} \left(\varphi \lor \psi \right) \to \left(\mathsf{Can} \, \varphi \lor \mathsf{Can} \, \psi \right)$$

 I can hit bottom or top, but NOT (I can hit bottom -or-I can hit top).

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).



$$\mathbf{0} \not\models \varphi \to \operatorname{\mathsf{Can}} \varphi$$

• I hit the bull, but I am not *able* to hit the bull.

イロト イポト イヨト イヨト

• I can hit bottom or top, but NOT (I can hit bottom -or-I can hit top).

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).



$$\mathbf{0} \not\models \varphi \to \operatorname{\mathsf{Can}} \varphi$$

• I hit the bull, but I am not *able* to hit the bull.

< ロト (周) (日) (日)

$$(\mathbf{2} \not\models \mathsf{Can} \, (\varphi \lor \psi) \to (\mathsf{Can} \, \varphi \lor \mathsf{Can} \, \psi)$$

• I can hit bottom or top, but NOT (I can hit bottom -or-I can hit top).

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).



$$\mathbf{0} \not\models \varphi \to \operatorname{\mathsf{Can}} \varphi$$

• I hit the bull, but I am not *able* to hit the bull.

$$(\mathbf{2} \not\models \mathsf{Can} \, (\varphi \lor \psi) \to (\mathsf{Can} \, \varphi \lor \mathsf{Can} \, \psi)$$

 I can hit bottom or top, but NOT (I can hit bottom -or-I can hit top).

(1) rules out strong modal logics.

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).



$$\mathbf{0} \not\models \varphi \to \operatorname{\mathsf{Can}} \varphi$$

• I hit the bull, but I am not *able* to hit the bull.

$$(\mathbf{2} \not\models \mathsf{Can} \, (\varphi \lor \psi) \to (\mathsf{Can} \, \varphi \lor \mathsf{Can} \, \psi)$$

 I can hit bottom or top, but NOT (I can hit bottom -or-I can hit top).

(2) rules out every Kripke model.

Ability is closely related to Means-end ascriptions. Modal logic cannot represent ability (Kenny).



$$\mathbf{0} \not\models \varphi \to \operatorname{\mathsf{Can}} \varphi$$

• I hit the bull, but I am not *able* to hit the bull.

< ロト (周) (日) (日)

- $(\mathbf{2} \not\models \mathsf{Can} \, (\varphi \lor \psi) \to (\mathsf{Can} \, \varphi \lor \mathsf{Can} \, \psi)$
 - I can hit bottom or top, but NOT (I can hit bottom -or-I can hit top).

(2) rules out *every* Kripke model. Trouble!

But not so fast...

Minimal models are weaker than Kripke semantics.

Minimal models

Relevance function: $\alpha : \mathcal{W} \to \mathcal{PPW}$

Intuitively: Each set S in $\alpha(w)$ is an action in w. If $S \models \varphi$, then doing S will make φ true.

イロト イヨト イヨト イヨト

But not so fast... Minimal models are weaker than Kripke semantics.

Minimal models Relevance function: $\alpha : W \to PPW$

Intuitively: Each set *S* in $\alpha(w)$ is an action in *w*. If $S \models \omega$, then doing *S* will make ω true.

(日) (部) (注) (注)

But not so fast... Minimal models are weaker than Kripke semantics.

Minimal models

Relevance function: $\alpha : \mathcal{W} \to \mathcal{PPW}$

 $w\models {\sf Can}\,arphi$ iff there is $S\in lpha(w)$ such that $S\models arphi$

Intuitively: Each set S in $\alpha(w)$ is an action in w. If $S \models \varphi$, then doing S will make φ true.

・ロト ・ 同ト ・ ヨト ・ ヨト

But not so fast... Minimal models are weaker than Kripke semantics.

Minimal models

Relevance function: $\alpha : \mathcal{W} \to \mathcal{PPW}$ $w \models \operatorname{Can} \varphi$ iff there is $S \in \alpha(w)$ such that $S \models \varphi$.

Intuitively: Each set S in $\alpha(w)$ is an action in w. If $S \models \varphi$, then doing S will make φ true.

・ロト ・ 同ト ・ ヨト ・ ヨト

But not so fast... Minimal models are weaker than Kripke semantics.

Minimal models

Relevance function: $\alpha : \mathcal{W} \to \mathcal{PPW}$ $w \models \operatorname{Can} \varphi$ iff there is $S \in \alpha(w)$ such that $S \models \varphi$.

Intuitively: Each set S in $\alpha(w)$ is an action in w. If $S \models \varphi$, then doing S will make φ true.

(D) (A) (A)

But not so fast... Minimal models are weaker than Kripke semantics.

Minimal models

Relevance function: $\alpha : \mathcal{W} \to \mathcal{PPW}$ $w \models \operatorname{Can} \varphi$ iff there is $S \in \alpha(w)$ such that $S \models \varphi$.

Intuitively: Each set S in $\alpha(w)$ is an action in w. If $S \models \varphi$, then doing S will make φ true.

(D) (A) (A)

Brown's ability logic is very closely related to our means-end logic.

There is a natural translation of dynamic logic to minimal models.

 $w \models \operatorname{Can} \varphi$ iff there is some *m* such that $w \models \langle m \rangle \varphi$.

One *can* make φ true iff he has a *means* to φ .

Actually, minimal models make sense for our actions too... but let's not complicate matters.

Brown's ability logic is very closely related to our means-end logic.

There is a natural translation of dynamic logic to minimal models.

 $w \models \operatorname{Can} \varphi$ iff there is some *m* such that $w \models \langle m \rangle \varphi$.

One *can* make φ true iff he has a *means* to φ .

Actually, minimal models make sense for our actions too... but let's not complicate matters.

Brown's ability logic is very closely related to our means-end logic.

There is a natural translation of dynamic logic to minimal models.

 $w \models \operatorname{Can} \varphi$ iff there is some *m* such that $w \models \langle m \rangle \varphi$.

One *can* make φ true iff he has a *means* to φ .

Actually, minimal models make sense for our actions too... but let's not complicate matters.

Brown's ability logic is very closely related to our means-end logic.

There is a natural translation of dynamic logic to minimal models.

 $w \models \operatorname{Can} \varphi$ iff there is some *m* such that $w \models \langle m \rangle \varphi$.

One *can* make φ true iff he has a *means* to φ .

Actually, minimal models make sense for our actions too... but let's not complicate matters.

・ロト ・ 同ト ・ ヨト ・ ヨト

Brown's ability logic is very closely related to our means-end logic.

There is a natural translation of dynamic logic to minimal models.

 $w \models \operatorname{Can} \varphi$ iff there is some *m* such that $w \models \langle m \rangle \varphi$.

One *can* make φ true iff he has a *means* to φ .

Actually, minimal models make sense for our actions too... but let's not complicate matters.

・ロト ・ 同ト ・ ヨト ・ ヨト

Brown's ability logic is very closely related to our means-end logic.

There is a natural translation of dynamic logic to minimal models.

 $w \models \operatorname{Can} \varphi$ iff there is some *m* such that $w \models \langle m \rangle \varphi$.

One *can* make φ true iff he has a *means* to φ .

Actually, minimal models make sense for our actions too... but let's not complicate matters.

Outline

Means and ends, informally

 Norms in Knowledge
 Initial analysis

 Means and ends, formally

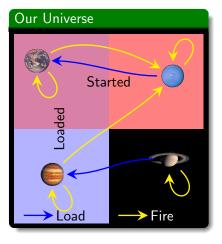
 Propositional Dynamic Logic
 Brown's Logic of Ability

- Means and ends, fuzzily
 - Why go fuzzy?
 - Fuzzy sets

< □> < □> < □>

Why go fuzzy? Fuzzy sets

Efficacy as an essential feature of means



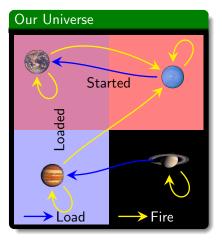
Our picture is unreasonable.

A misfire is less likely than a retort.

We should add probabilities to the picture.

Why go fuzzy? Fuzzy sets

Efficacy as an essential feature of means



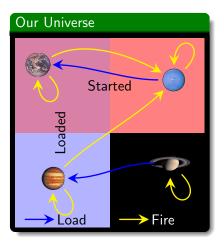
Our picture is unreasonable. A misfire is less likely than a retort.

We should add probabilities to the picture.

- ・ 同 ト・ ・ ヨ ト

Why go fuzzy? Fuzzy sets

Efficacy as an essential feature of means



Our picture is unreasonable.

A misfire is less likely than a retort.

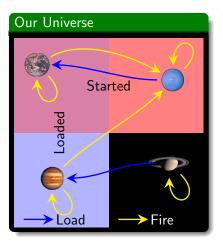
We should add probabilities to the picture.

A (1) > A (2) > A

3.0

Why go fuzzy? Fuzzy sets

Efficacy as an essential feature of means



Our picture is unreasonable.

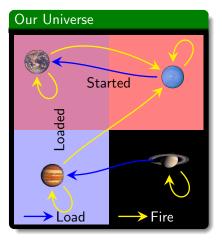
A misfire is less likely than a retort.

We should add probabilities to the picture.

▲□ ▶ ▲ □ ▶ ▲

Why go fuzzy? Fuzzy sets

A fuzzy approach



The need for probabilities goes deeper than this.

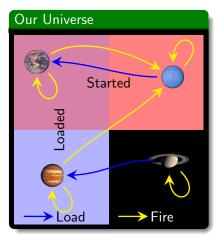
Different means to same end have different efficacies.

We add probabilities to our transitions... but that's only part of the solution.

< ロト (周) (日) (日)

Why go fuzzy? Fuzzy sets

A fuzzy approach



The need for probabilities goes deeper than this.

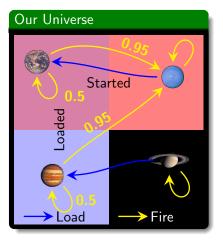
Different means to same end have different efficacies.

We add probabilities to our transitions... but that's only part of the solution.

(4月) (4日) (4日)

Why go fuzzy? Fuzzy sets

A fuzzy approach



The need for probabilities goes deeper than this.

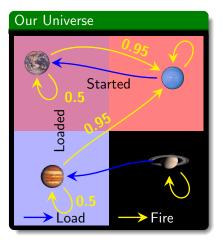
Different means to same end have different efficacies.

We add probabilities to our transitions... but that's only part of the solution.

・ 同 ト・ イヨ ト・ イヨ ト

Why go fuzzy? Fuzzy sets

A fuzzy approach



The need for probabilities goes deeper than this.

Different means to same end have different efficacies.

We add probabilities to our transitions... but that's only part of the solution.

▲□ ▶ ▲ □ ▶ ▲

Informal Formal Fuzzy Why go fuzzy? Fuzzy sets

A brief introduction to fuzzy set theory



In God's set theory, the membership relation is two-valued.

Each x is either in S or not.

But for mere mortals...

イロト イポト イヨト イヨト

Informal Formal Fuzzy Fuzzy sets

A brief introduction to fuzzy set theory



In God's set theory, the membership relation is two-valued.

Each x is either in S or not.

But for mere mortals...

- ・ 同 ト・ ・ ヨ ト

Informal Formal Fuzzy Fuzzy sets

A brief introduction to fuzzy set theory



In God's set theory, the membership relation is two-valued.

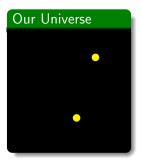
Each x is either in S or not.

But for mere mortals...

・ 同 ト・ ・ ヨート・ ・ ヨート

Informal Formal Fuzzy Why go fuzzy? Fuzzy sets

A brief introduction to fuzzy set theory



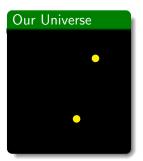
Some propositions aren't so *crisp*.

Fuzzy sets represent ambiguous propositions.

Here, $x \in S$ is assigned some value in [0, 1].

Informal Formal Fuzzy Fuzzy sets

A brief introduction to fuzzy set theory

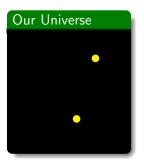


Some propositions aren't so *crisp*. Fuzzy sets represent ambiguous propositions.

Here, $x \in S$ is assigned some value in [0, 1].

- ・ 同 ト・ ・ ヨ ト

A brief introduction to fuzzy set theory



Some propositions aren't so *crisp*.

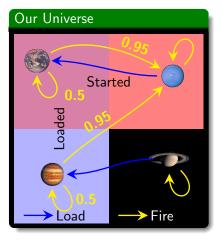
Fuzzy sets represent ambiguous propositions.

Here, $x \in S$ is assigned some value in [0, 1].

A (1) > A (2) > A

Why go fuzzy? Fuzzy sets

A fuzzy approach



Think again about [fire]**Started**.

That is neither just true nor false.

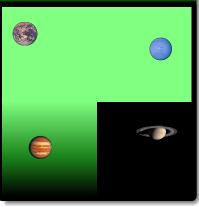
< □ > < □ > < □ >

It's a bit fuzzy.

Why go fuzzy? Fuzzy sets

A fuzzy approach

Our Universe



Think again about [fire]**Started**.

That is neither just true nor false.

< □ > < □ > < □ >

æ

lt's a bit fuzzy.

Why go fuzzy? Fuzzy sets

A fuzzy approach

Our Universe



Think again about [fire]**Started**.

That is neither just true nor false.

< □ > < □ > < □ >

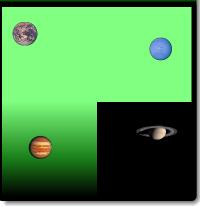
æ

It's a bit fuzzy.

Why go fuzzy? Fuzzy sets

A fuzzy approach

Our Universe



Now, this is a new approach.

There *are* fuzzy modal logics...but they're different.

Our fuzzy dynamic logic uses expected values, not conjunctions of implications.

(日) (部) (注) (注)

Why go fuzzy? Fuzzy sets

A fuzzy approach

Our Universe

9	

Now, this is a new approach.

There *are* fuzzy modal logics...but they're different.

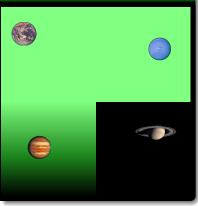
Our fuzzy dynamic logic uses expected values, not conjunctions of implications.

・ 同 ト・ ・ ヨート・ ・ ヨート

Why go fuzzy? Fuzzy sets

A fuzzy approach

Our Universe



Now, this is a new approach.

There *are* fuzzy modal logics...but they're different.

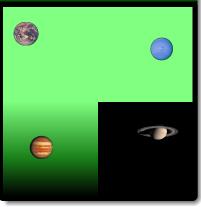
Our fuzzy dynamic logic uses expected values, not conjunctions of implications.

・ 同・ ・ ヨ・・・・

Why go fuzzy? Fuzzy sets

A fuzzy approach

Our Universe



Now, this is a new approach.

There *are* fuzzy modal logics...but they're different.

Our fuzzy dynamic logic uses expected values, not conjunctions of implications.

▲□ ▶ ▲ □ ▶ ▲

Why go fuzzy? Fuzzy sets

The Main Issues

• The relationship between ability and means.

- Fuzzy sets and dynamic logic.
- Conditions and means-chaining.
- Back to artifactual functions.

(日) (部) (注) (注)

æ

The Main Issues

- The relationship between ability and means.
- Fuzzy sets and dynamic logic.
- Conditions and means-chaining.
- Back to artifactual functions.

-2

The Main Issues

- The relationship between ability and means.
- Fuzzy sets and dynamic logic.
- Conditions and means-chaining.
- Back to artifactual functions.

・ 同 ト・ イヨ ト・ イヨ ト

-2

Why go fuzzy? Fuzzy sets

The Main Issues

- The relationship between ability and means.
- Fuzzy sets and dynamic logic.
- Conditions and means-chaining.
- Back to artifactual functions.

▲□▶ ▲ □▶ ▲ □▶

-2