The Muddy Children: A logic for public announcement

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Eindhoven University of Technology

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Outline



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Outline





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2 Modal logics

3 The epistemic operator

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2 Modal logics

- 3 The epistemic operator
- A logic for public announcement

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Outline



2 Modal logics

- 3 The epistemic operator
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The muddy children



Quincy

Hughes The Muddy Children: A logic for public announcement

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The muddy children



Quincy



Prescott

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The muddy children



Quincy



Prescott



Hughes

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The muddy children



Baba: "At least one of you is muddy."

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The muddy children



Baba: "At least one of you is muddy." Baba: "Are you muddy?"

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The muddy children



Baba: "At least one of you is muddy." Baba: "Are you muddy?" Quincy: "I don't know." Prescott: "I don't know." Hughes: "I don't know."

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The muddy children



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The muddy children



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The muddy children



Baba: "At least one of you is muddy." Baba: "Are you muddy?" Quincy: "I don't know." Prescott: "I don't know." Hughes: "I don't know."

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The muddy children



Baba: "Are you muddy?"

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The muddy children



Baba: "Are you muddy?" Quincy: "Aha! What if I <u>wasn't</u> muddy?"

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The muddy children



Baba: "Are you muddy?" Quincy: "Aha! What if I <u>wasn't</u> muddy?" Quincy: "Then Prescott would not have seen any muddy kids."

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The muddy children



- Baba: "Are you muddy?"
- Quincy: "Aha! What if I wasn't muddy?"
- Quincy: "Then Prescott would not have seen any muddy kids."
- Quincy: "Prescott would have said 'yes' last time!"

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The muddy children



Baba: "Are you muddy?"

- Quincy: "Aha! What if I wasn't muddy?"
- Quincy: "Then Prescott would not have seen any muddy kids."
- Quincy: "Prescott would have said 'yes' last time!"

Quincy: "I must be muddy."

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The muddy children









Hughes

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Quincy

Prescott

Baba: "Are you muddy?" Quincy: "Yes." Prescott: "Yes." Hughes: "I don't know."

The muddy children



When Baba said, "At least one kid is muddy," every kid knew that...

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The muddy children



When Baba said, "At least one kid is muddy," every kid knew that... <u>but</u> they didn't know that the other kids knew that!

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The muddy children



When Baba said, "At least one kid is muddy," every kid knew that... <u>but</u> they didn't know that the other kids knew that!

Public announcements of φ tell you φ , everyone knows φ , everyone knows that everyone knows φ , ...

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Outline

The muddy children

2 Modal logics

- 3 The epistemic operator
- 4 A logic for public announcement

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Modal operators

A modal operator \Box is a logical operator.

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Modal operators

A modal operator \Box is a logical operator. We use it to build new formulas from old.

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Modal operators

A <u>modal operator</u> \Box is a <u>logical operator</u>. We use it to build new formulas from old. If φ is a formula, then so is $\Box \varphi$.

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Modal operators

A modal operator \Box is a logical operator.

We use it to build new formulas from old.

If φ is a formula, then so is $\Box \varphi$.

We use modal operators to express lots of concepts, including: Necessarily φ . $\Box \varphi$

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 φ will always be true. $G\varphi$

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- A modal operator \square is a logical operator.
- We use it to build new formulas from old.
- If φ is a formula, then so is $\Box \varphi$.
- We use modal operators to express lots of concepts, including:
- Necessarily φ . $\Box \varphi$ φ will always be true. $G \varphi$
- φ is provable. Prov φ

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 - It ought to be φ . $O\varphi$

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- Necessarily φ . $\Box \varphi$ $\Diamond \varphi$ Possibly φ . φ will always be true. $G \varphi$
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Each operator \Box has a dual, $\Diamond = \neg \Box \neg$.

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 - Necessarily φ . $\Box \varphi$ $\Diamond \varphi$ Possibly φ .
 - φ will always be true. $G\varphi$ $F\varphi$ Eventually φ .
- Prov φ ?? φ is not refutable. φ is provable.
- It ought to be φ . $O\varphi$
- $K\varphi$ I know φ .

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 - φ is provable. Prov φ ?? φ is not refutable.
 - It ought to be φ . $O\varphi$ $P\varphi$ φ is permitted. I know φ . $K\varphi$

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Modal operators

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- We use modal operators to express lots of concepts, including:
 - Necessarily φ . $\Box \varphi \qquad \Diamond \varphi$ Possibly φ .
 - φ will always be true. $G\varphi$ $F\varphi$ Eventually φ .
 - φ is provable. Prov φ ?? φ is not refutable.
 - It ought to be φ . $O\varphi$ $P\varphi$ φ is permitted.
 - I know φ . $K\varphi$?? I think φ is possible.

Each operator \Box has a dual, $\Diamond = \neg \Box \neg$.

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Kripke semantics

Models for modal logics are based on "possible world" semantics.

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Kripke semantics

Models for modal logics are based on "possible world" semantics. Let \mathcal{W} be a set of worlds with a graph.



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Kripke semantics

Models for modal logics are based on "possible world" semantics. Let \mathcal{W} be a set of worlds with a graph.

Write $w \models P$ if P is true at world w.



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Kripke semantics

Models for modal logics are based on "possible world" semantics. Let $\mathcal W$ be a set of worlds with a graph.

Write $w \models P$ if P is true at world w.

$$\begin{array}{ll} w \models \varphi \land \psi & \text{iff} & w \models \varphi \text{ and } w \models \psi \\ w \models \varphi \lor \psi & \text{iff} & w \models \varphi \text{ or } w \models \psi \\ w \models \varphi \rightarrow \psi & \text{iff} & w \models \psi \text{ or } w \not\models \varphi \\ w \models \neg \varphi & \text{iff} & w \not\models \varphi \end{array}$$



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Kripke semantics

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$$w \models \varphi \rightarrow \psi \quad \text{iff} \quad w \models \psi \text{ or } w \not\models \varphi$$
$$w \models \neg \varphi \quad \text{iff} \quad w \not\models \varphi$$
$$w \models \neg \varphi \quad \text{iff for every } w \longrightarrow w',$$

 $w' \models \varphi$.



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Kripke semantics

Models for modal logics are based on "possible world" semantics. Let $\mathcal W$ be a set of worlds with a graph.

Write $w \models P$ if P is true at world w.

$$w \models \varphi \land \psi \quad \text{iff} \quad w \models \varphi \text{ and } w \models \psi$$

$$w \models \varphi \lor \psi \quad \text{iff} \quad w \models \varphi \text{ or } w \models \psi$$

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$$w \models \neg \varphi \quad \text{iff} \quad w \not\models \varphi$$

$$w \models \Box \varphi \text{ iff for every } w \longrightarrow w',$$

$$w' \models \varphi.$$

$$w \models \Diamond \varphi \text{ iff there is } w \longrightarrow w'$$
such that $w' \models \varphi$.



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Modal axioms and frame conditions

Axioms on \Box correspond to conditions on the graph.

NameAxiomGraph is...(D) $\Box \varphi \rightarrow \Diamond \varphi$ serial



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Modal axioms and frame conditions

Axioms on \Box correspond to conditions on the graph.

NameAxiomGraph is...(D) $\Box \varphi \rightarrow \Diamond \varphi$ serial(M) $\Box \varphi \rightarrow \varphi$ reflexive



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Modal axioms and frame conditions

Axioms on \Box correspond to conditions on the graph.

NameAxiomGraph is...(D) $\Box \varphi \rightarrow \Diamond \varphi$ serial(M) $\Box \varphi \rightarrow \varphi$ reflexive(4) $\Box \varphi \rightarrow \Box \Box \varphi$ transitive



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Modal axioms and frame conditions

Axioms on \Box correspond to conditions on the graph.

NameAxiomGraph is...(D) $\Box \varphi \rightarrow \Diamond \varphi$ serial(M) $\Box \varphi \rightarrow \varphi$ reflexive(4) $\Box \varphi \rightarrow \Box \Box \varphi$ transitive(B) $\varphi \rightarrow \Box \Diamond \varphi$ symmetric



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Modal axioms and frame conditions

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Name Axiom Graph is... (D) $\Box \varphi \to \Diamond \varphi$ serial (M) $\Box \varphi \to \varphi$ reflexive (4) $\Box \varphi \to \Box \Box \varphi$ transitive (B) $\varphi \to \Box \Diamond \varphi$ symmetric $\Diamond \varphi \to \Box \Diamond \varphi$ (5)euclidean



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Modal axioms and frame conditions

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Name Axiom Graph is... (D) $\Box \varphi \to \Diamond \varphi$ serial $\Box \varphi \to \varphi$ (M) reflexive (4) $\Box \varphi \to \Box \Box \varphi$ transitive (B) $\varphi \to \Box \Diamond \varphi$ symmetric (5) $\Diamond \varphi \to \Box \Diamond \varphi$ euclidean

If \Box satisfies (M), (4) and (B), then the graph is an equivalence relation.



Modal axioms and frame conditions

Axioms on \Box correspond to conditions on the graph.

NameAxiomGraph is...(D) $\Box \varphi \rightarrow \Diamond \varphi$ serial(M) $\Box \varphi \rightarrow \varphi$ reflexive(4) $\Box \varphi \rightarrow \Box \Box \varphi$ transitive(B) $\varphi \rightarrow \Box \Diamond \varphi$ symmetric(5) $\Diamond \varphi \rightarrow \Box \Diamond \varphi$ euclidean

If \Box satisfies (M), (4) and (B), then the graph is an equivalence relation.

reflexive transitive symmetric euclidean

serial

• Write w - w'.

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Modal axioms and frame conditions

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NameAxiomGraph is...(D) $\Box \varphi \rightarrow \Diamond \varphi$ serial(M) $\Box \varphi \rightarrow \varphi$ reflexive(4) $\Box \varphi \rightarrow \Box \Box \varphi$ transitive(B) $\varphi \rightarrow \Box \Diamond \varphi$ symmetric(5) $\Diamond \varphi \rightarrow \Box \Diamond \varphi$ euclidean

If \Box satisfies (M), (4) and (B), then the graph is an equivalence relation.

- Write *w w*'.
- Don't bother to draw loops.



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Outline

The muddy children

2 Modal logics

- 3 The epistemic operator
- A logic for public announcement

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The epistemic operator



For each agent α , we introduce an operator K_{α} .

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The epistemic operator



For each agent α , we introduce an operator K_{α} . $K_{\alpha}\varphi$ means " α knows φ ."

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The epistemic operator



For each agent α , we introduce an operator K_{α} . $K_{\alpha}\varphi$ means " α knows φ ." Each α has its own graph, too.

The epistemic operator



For each agent α , we introduce an operator K_{α} . $K_{\alpha}\varphi$ means " α knows φ ." Each α has its own graph, too. An edge $w - \frac{\alpha}{2} w'$ means " α can not distinguish w from w'."

The epistemic operator



For each agent α , we introduce an operator K_{α} . $K_{\alpha}\varphi \text{ means } ``\alpha \text{ knows } \varphi.''$ Each α has its own graph, too. An edge $w \xrightarrow{\alpha} w' \text{ means } ``\alpha \text{ can not distinguish } w \text{ from } w'.''$ $w \models K_{\alpha}\varphi \text{ iff for every } w \xrightarrow{\alpha} w', w' \models \varphi.$

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More on K_{α}



 $K_{\alpha}\varphi$ means " α knows φ ."

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More on K_{α}



 $K_{\alpha}\varphi$ means " α knows φ ." What does $\neg K_{\alpha}\neg\varphi$ mean?

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More on K_{α}



 $K_{\alpha}\varphi$ means " α knows φ ." What does $\neg K_{\alpha}\neg\varphi$ mean? α considers that φ is possible.

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More on K_{α}



 $K_{\alpha}\varphi$ means " α knows φ ." What does $\neg K_{\alpha}\neg\varphi$ mean? α considers that φ is possible. What about $K_{\alpha}K_{\beta}\varphi$?

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More on K_{α}



 $K_{\alpha}\varphi$ means " α knows φ ." What does $\neg K_{\alpha}\neg\varphi$ mean? α considers that φ is possible. What about $K_{\alpha}K_{\beta}\varphi$? α knows that β knows that φ .

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More on K_{α}



 $K_{\alpha}\varphi \text{ means}$ " α knows φ ." What does $\neg K_{\alpha}\neg\varphi$ mean? α considers that φ is possible. What about $K_{\alpha}K_{\beta}\varphi$? α knows that β knows that φ . For instance, Quincy knows that Hughes knows that Prescott is muddy.

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More on K_{α}



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Properties of K_{α}



Properties of K_{α}







Quincy

Prescott

Hughes

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 $\begin{array}{c} {\sf K}_{\alpha}\varphi \to \varphi \\ {\sf K}_{\alpha}\varphi \to {\sf K}_{\alpha}{\sf K}_{\alpha}\varphi \end{array}$

knowledge is true positive introspection

Properties of K_{α}







Quincy

Prescott

Hughes

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knowledge is true positive introspection negative introspection

Properties of K_{α}







Quincy

Prescott

Hughes

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 $\begin{array}{c} \mathsf{K}_{\alpha}\varphi \to \varphi \\ \mathsf{K}_{\alpha}\varphi \to \mathsf{K}_{\alpha}\mathsf{K}_{\alpha}\varphi \\ \neg \mathsf{K}_{\alpha}\varphi \to \mathsf{K}_{\alpha}\neg \mathsf{K}_{\alpha}\varphi \\ \mathsf{K}_{\alpha}(\varphi \to \psi) \to (\mathsf{K}_{\alpha}\varphi \to \mathsf{K}_{\alpha}\psi) \end{array}$

knowledge is true positive introspection negative introspection distributivity

Universal and common knowledge



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Universal and common knowledge

- Universal knowledge $(E\varphi)$:
 - Everyone knows φ .
 - No one-step paths outside of φ .



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Universal and common knowledge

• Universal knowledge $(E\varphi)$:

- Everyone knows φ .
- No one-step paths outside of φ .
- Universal knowledge of universal knowledge (*EE*φ):
 - Everyone knows that everyone knows φ .
 - No two-step paths outside of φ .
 - No one-step paths outside of universal knowledge.


Universal and common knowledge

• Universal knowledge $(E\varphi)$:

- Everyone knows φ .
- No one-step paths outside of φ .
- Universal knowledge of universal knowledge (*EE*φ):
 - Everyone knows that everyone knows φ .
 - No two-step paths outside of φ .
 - No one-step paths outside of universal knowledge.
- Common knowledge $(C\varphi)$:
 - Everyone knows that everyone knows that...that everyone knows φ .
 - No paths out of φ .



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Back to the kids

Eight possible worlds.

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Back to the kids

Eight possible worlds.

- 0 clean
- 1 muddy

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Back to the kids



Eight possible worlds.

- 0 clean
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Back to the kids



Eight possible worlds.

- 0 clean
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Back to the kids

Eight possible worlds.

- 0 clean
- 1 muddy



Back to the kids



Quincy is muddy in these four worlds.

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Back to the kids



Quincy is muddy in these four worlds. Prescott is muddy in these four.

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Back to the kids



Quincy is muddy in these four worlds.

Prescott is muddy in these four. And Hughes is muddy in these four.

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Back to the kids



Quincy is muddy in these four worlds.

Prescott is muddy in these four. And Hughes is muddy in these four.

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Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't.

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Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't. World 110 is indistinguishable from 010.

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Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't. World 110 is indistinguishable from 010. Quincy's epistemic relation.

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Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't. World 110 is indistinguishable from 010. Quincy's epistemic relation. Prescott's relation.

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Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't. World 110 is indistinguishable from 010. Quincy's epistemic relation. Prescott's relation. And Hughes's relation.

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Back to the kids



Quincy cannot distinguish a world where he is muddy from one where he isn't. World 110 is indistinguishable from 010. Quincy's epistemic relation. Prescott's relation. And Hughes's relation.

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Outline

The muddy children

Modal logics

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- A logic for public announcement

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Dynamic features



What happens when someone announces φ ?

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Dynamic features



What happens when someone announces φ ?

Everyone learns that φ was true when announced.

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Dynamic features



What happens when someone announces φ ?

Everyone learns that φ was true when announced.

So the $\neg \varphi$ worlds are unimportant. Take 'em out!

3 ×

Dynamic features



What happens when someone announces φ ?

Everyone learns that φ was true when announced.

So the $\neg \varphi$ worlds are unimportant. Take 'em out! Edges, too!

3 ×

Dynamic features



What happens when someone announces φ ?

Everyone learns that φ was true when announced.

So the $\neg \varphi$ worlds are unimportant. Take 'em out! Edges, too!

Information reduces uncertainty by eliminating possibilities.

A model of possible models!



• Announcing φ changes the model.

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A model of possible models!



- Announcing φ changes the model.
- Announcing ψ changes it another way.

A model of possible models!



- Announcing φ changes the model.
- Announcing ψ changes it another way.
- Get a transition system on models.

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A model of possible models!



- Announcing φ changes the model.
- Announcing ψ changes it another way.
- Get a transition system on models.
- Another Kripke frame!

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Resolving the muddy children



Baba: "At least one of you is muddy."

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Resolving the muddy children



Baba: "At least one of you is muddy." World 000 is inconsistent with this announcement.

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Resolving the muddy children



Baba: "At least one of you is muddy." World 000 is inconsistent with this announcement. We remove it from the model.

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Resolving the muddy children



Baba: "At least one of you is muddy." World 000 is inconsistent with this announcement. We remove it from the model. Before $w_{110} \models E\varphi$.

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Resolving the muddy children



Baba: "At least one of you is muddy." World 000 is inconsistent with this announcement. We remove it from the model. Before $w_{110} \models E\varphi$. Now $w_{110} \models C\varphi$.

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Resolving the muddy children



Baba: "Are you muddy?" Quincy: "I don't know." Prescott: "I don't know." Hughes: "I don't know."

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Resolving the muddy children



Baba: "Are you muddy?" Quincy: "I don't know." Prescott: "I don't know." Hughes: "I don't know."

Remove world 100!

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Resolving the muddy children



Baba: "Are you muddy?" Quincy: "I don't know." Prescott: "I don't know." Hughes: "I don't know."

Remove world 100! Remove world 010!

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Resolving the muddy children



Baba: "Are you muddy?" Quincy: "I don't know." Prescott: "I don't know." Hughes: "I don't know."

Remove world 100! Remove world 010! Remove world 001!

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Resolving the muddy children



Baba: "Are you muddy?" Quincy: "I don't know." Prescott: "I don't know." Hughes: "I don't know."

Remove world 100! Remove world 010! Remove world 001!

A much simpler model!

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Resolving the muddy children



But now: $w_{110} \models K_Q("Q \text{ is muddy}")!$

Hughes The Muddy Children: A logic for public announcement

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Resolving the muddy children



But now: $w_{110} \models K_Q$ ("Q is muddy")! Baba: "Are you muddy?" Quincy: "Yes!" Prescott: "Yes!" Hughes: "I don't know."

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Resolving the muddy children



But now: $w_{110} \models K_Q$ ("Q is muddy")! Baba: "Are you muddy?" Quincy: "Yes!" Prescott: "Yes!" Hughes: "I don't know."

Quincy knows Quincy is muddy: remove 011 and 111.

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Resolving the muddy children



But now: $w_{110} \models K_Q$ ("*Q* is muddy")! Baba: "Are you muddy?" Quincy: "Yes!" Prescott: "Yes!" Hughes: "I don't know." Quincy knows Quincy is mude

Quincy knows Quincy is muddy: remove 011 and 111.

Prescott knows Prescott is muddy: remove 111 and 101.

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Resolving the muddy children



Baba: "Are you muddy?" Quincy: "Yes!" Prescott: "Yes!" Hughes: "No!"

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