Means-end Relations and a Measure of Efficacy

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July 4, 2005

Outline

1. Means-end relations
   - Interest I: Practical syllogisms
   - Interest II: Functional ascriptions
   - Propositional Dynamic Logic

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   - Interest I: Practical syllogisms
   - Interest II: Functional ascriptions
   - Propositional Dynamic Logic

2. Efficacy via fuzzy logic
   - Reliability as a fuzzy operator
   - The resulting fuzzy logic

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Means-end relations in practical syllogisms

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This premise is a *means-end relation*.

An example from von Wright

I want to make the hut habitable.

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But *necessary* means-end relations are a bit tricky.

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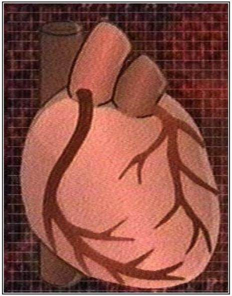
I want to make the hut habitable.
If I heat the hut, it will be habitable.

Therefore, I have reason to heat the hut.

An alternative with a sufficient means-end relation.

Functional ascriptions



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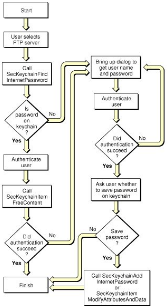

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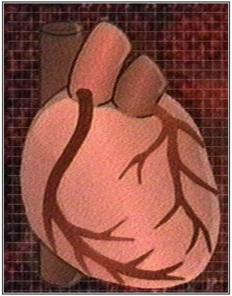

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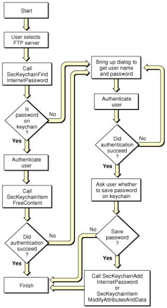

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  - distinct of desirability
  - distinct from theory of practical reasoning

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- Objects as means?

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Propositional Dynamic Logic is a logic of actions.

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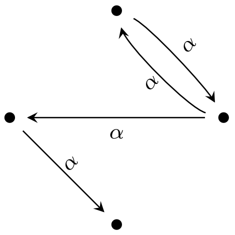
- $\langle\alpha\rangle\varphi$: after doing α, φ *might* hold.

PDL semantics



Possible world semantics with transition systems for each action α.

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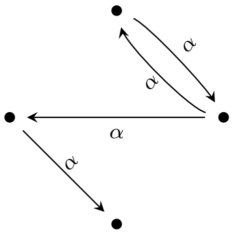

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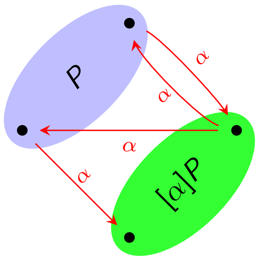

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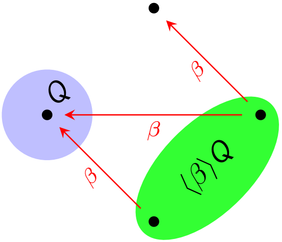

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$$w \models \langle\alpha\rangle\varphi \;\; \textit{iff} \;\; \exists\, w \xrightarrow{\alpha} w' \;.\; w' \models \varphi.$$

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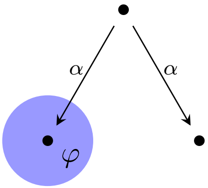
Two interpretations:



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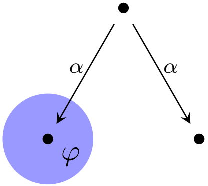

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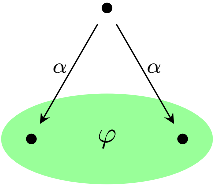

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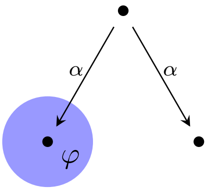

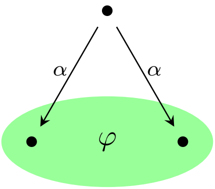

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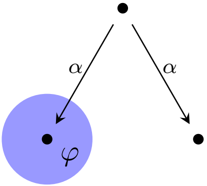

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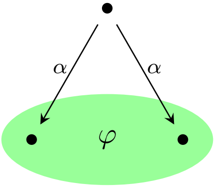

Strong: α *will* realize φ.

$w \models [\alpha]\varphi \wedge \underbrace{\langle\alpha\rangle\top}_{\alpha \text{ can be done.}}$

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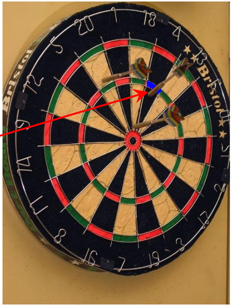

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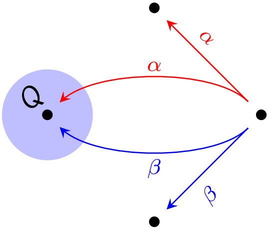
Efficacy: The degree of reliability of a means to an end.

From non-determinism to probabilities



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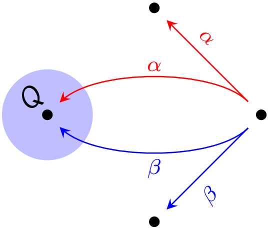

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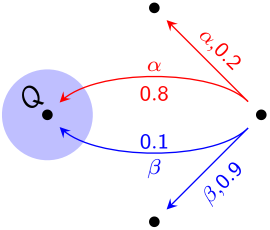

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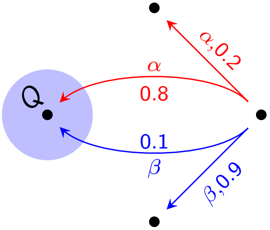

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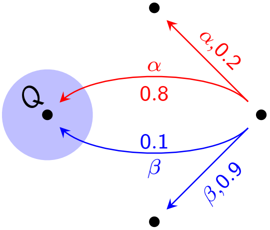

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Write: $P(\ w \xrightarrow{\alpha} w'\ ) = x$.

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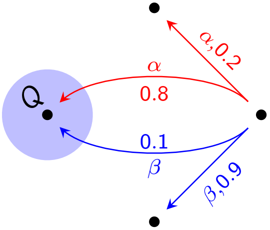

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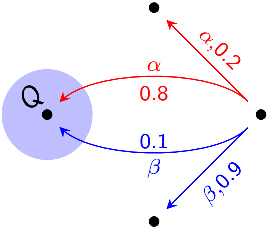

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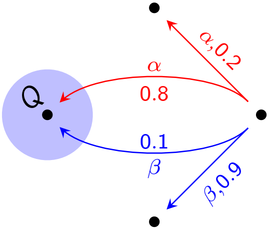

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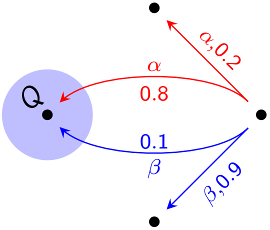

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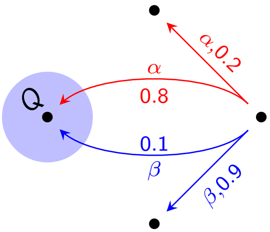

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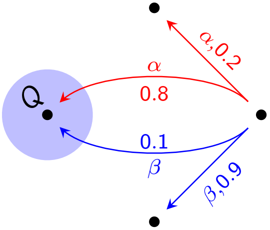

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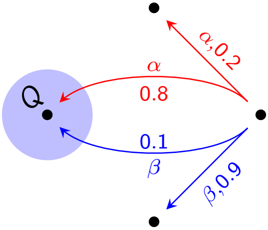

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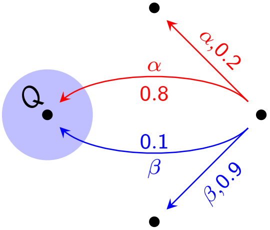

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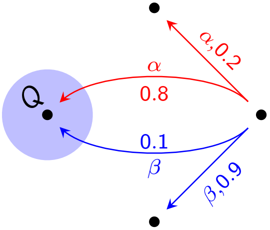

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Slogan: Probabilities and fuzziness are different.

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But probability $\neq$ fuzziness...

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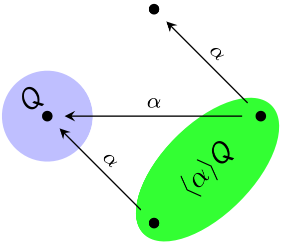
Truth degree of "Probably φ" $= P(\varphi)$.

Reliability as a fuzzy proposition

"Reliably", like "Probably", is a vague operator.



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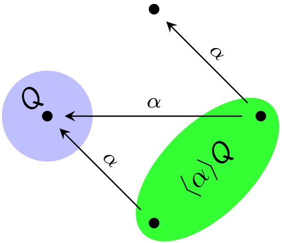

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$\langle\alpha\rangle\varphi \Leftrightarrow \alpha$ will possibly realize φ

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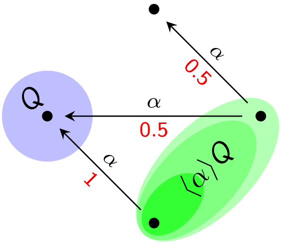

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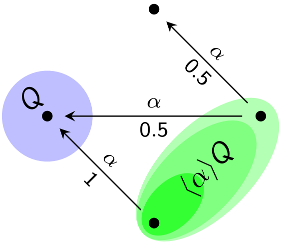

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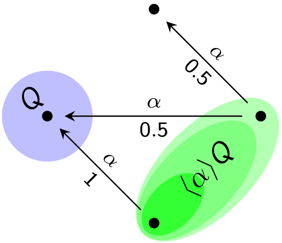

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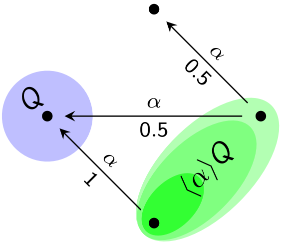

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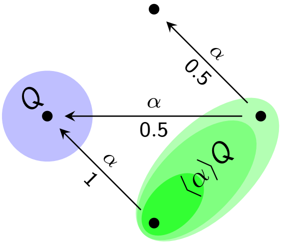

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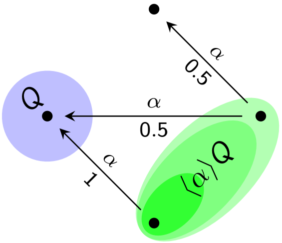

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- Elegant treatment of complex ends, like $\langle\alpha\rangle\varphi \wedge \langle\beta\rangle\psi$.

Fuzzy ends

An accidental advantage

Weapons are for causing harm.

Fuzzy ends
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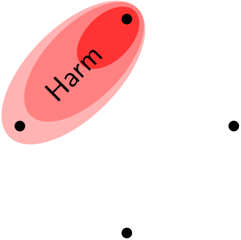

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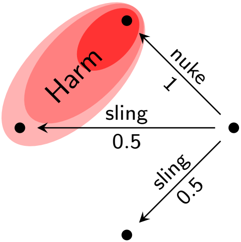

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This end is fuzzy.

Fuzzy PDL allows for fuzzy ends.

A nuke is more effective in causing harm than a slingshot.

(Duh.)

Extending the logic to other connectives

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The semantics of fuzzy PDL

On formulas

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Logical properties
Completeness

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I wish.

Logical properties
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But not with these semantics.

Ongoing work. . .

Concluding remarks

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Thank you.

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